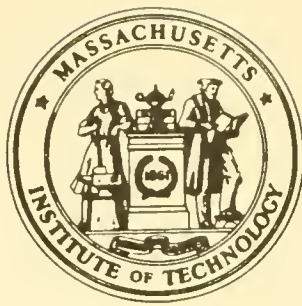




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Optimizing Consumer Advertising,
Intermediary Advertising and
Markup in a Vertical Market Structure*

474-70

Philippe A. Naert **

August 1970

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*Based on Chapter 3 of the author's Ph.D. dissertation.

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ABSTRACT

Given is a vertical market structure (VMS) which consists of producers, one intermediary level and consumers. The intermediary firms are assumed to be independent of the producers. Demand depends on the number of intermediaries, consumer advertising and intermediary advertising. In this paper behavior of intermediaries is explicitly taken into account in constructing optimization models for producers. Two models are presented: In the first the objective of the producers is to maximize sales, and in the second to maximize profit. The decision variables are consumer advertising, intermediary advertising, markup offered by producers to intermediaries. Behavior of intermediaries is incorporated through the use of a pseudo decision variable, the equilibrium number of middlemen in the VMS.

1. Introduction

Consider a VMS consisting of three levels, the producers (M), the intermediary level (W), and the consumers (N). The intermediaries will have two main functions: first of all, they will perform the communication in the VMS, and secondly, they will improve the availability of the product. By performing the communication function is meant that price information, orders etc. are processed through intermediaries rather than directly between producers and final consumers. The intermediary level will exist if entry of the first middleman results in a reduction of communication costs in the VMS. This is the case considered by Balderston [1], and extended by Baligh and Richartz [3] [4].¹ These models do not allow for direct communication between producers and consumers once intermediaries exist in the VMS. Here a direct link between producers and consumers is restored through consumer advertising. Also a persuasive element is added to the communication between producers and intermediaries. Demand will be assumed to depend on the availability of the product (which is related to the number of intermediaries), on consumer advertising and on advertising directed toward intermediaries, here called intermediary advertising. The assumptions are spelled out in section 2. The simple M-N structure is examined in section 3. A profit function for the producing level is introduced, so that in

later sections specific conditions for cooperation between level M and W can be stated. Conditions for cooperation are needed because middlemen are assumed to be independent from producers. The M-W-N structure without advertising is the subject of section 4. The results of section 4 will be used in section 5 where consumer advertising and intermediary advertising are introduced. Two models are then presented. In one the objective of producers is to maximize sales; in the other the objective is to maximize profit. The behavior of intermediaries is explicitly taken into account. Markup is also introduced as a decision variable, something that has received very little attention in the marketing literature. In section 6 an example is presented, and in section 7 some possible extensions are discussed.

2. Assumptions

- (I) The level of producers is called M. m , the number of producers in level M is given and constant. All firms are equal. N is the level of ultimate consumers, and n is the number of potential consumers. n is also assumed given and constant.
- (II) There is no product differentiation and consumers have no company or brand preferences.
- (III) All units in a given level are in contact with all units in adjacent levels.

(IV) When there are no intermediaries, the cost of communication incurred by producers is k per unit of time and per link.

(V) Suppose now that there is one intermediary level W . Consumers will now buy from that level and not from producers. We also assume that no more than one intermediary level can exist. Communication cost is k_1 per unit of time for each link $M-W$ and k_2 for each link $W-N$. k_1 and k_2 are not necessarily constant. They may for example depend on the number of intermediaries [11, Chapter 4]. Here we will assume that k_1 and k_2 are constant. k_3 is the amount spent by each producer on consumer advertising; k_4 is the amount spent by each producer on intermediary advertising (per intermediary).

(VI) The objective of producers will be joint profit or joint saler maximization. Given assumption (II) and assuming that it still holds when producers advertise, we can say that advertising aims at increasing sales rather than market share. Given this and assumption (I), joint profit or saler maximization and equal advertising expenditures for each firm reflects rational behavior and does not imply collusion.

- (VII) Required investment outlay for each intermediary entering level W is F . In fact it could be argued that investment outlays consist of a fixed amount and an amount related to the quantity transacted by each intermediary [11, Chapter 4]. Here the latter amount is assumed to be zero. Minimum acceptable rate of return on investment is r , so that $r F$ represents a normal profit.
- (VIII) Intermediaries do not incur inventory cost. We present a few examples where this assumption would hold. One is Balderston and Hoggatt's study of the lumber industry on the West Coast [2], where the physical product does not pass through the intermediaries. Another example is the catalogue ordering company which has outlets where the consumer can see some of the items on display, or where he can obtain additional information about a particular product and can place orders, but where the orders have to be filled from a warehouse.
- (IX) Commodity flow is Q per unit of time in the $M-N$ structure. In the $M-W-N$ structure without advertising, Q is a function of w , the number of firms in level W . In the $M-W-N$ structure with advertising, Q is a function of w , k_3 , k_4 . Note that demand is assumed not to depend on markup. This implies that intermediaries have to put the product on their shelves because consumers want it, whether the intermediaries like it or not.

(X) Price to consumers is assumed to be p , whether intermediaries exist or not. In the sales maximization case we will assume that producers are willing to cooperate with intermediaries as long as their profit is not reduced. That is producers do not require a rebate.² This condition for cooperation will not hold when profit maximization is the producer's objective. When necessary in section 5, the condition for cooperation will be modified.

3. M - N Structure

Let us first consider a VMS with only two levels: producers and consumers. Given assumption (III) the number of links in the M-N structure is mn . Let a be the number of units sold per communication link. a is then

$$(1) \quad a = Q/mn$$

Let π be profit for the m producers (excluding any allocated cost for overhead absorption):

$$(2) \quad \pi = Q(p - c_1 - c_2)$$

where,

p = unit market price

c_1 = average unit variable cost of production

c_2 = average unit variable cost of communication

We assume that p , c_1 and c_2 are constant. c_2Q is the total cost of communication per unit of time. Given that communication cost per link is k , and that there are mn links, total cost of communication is also equal to kmn . Therefore, we can replace c_2Q by kmn . From (1) it follows

that $Q = amn$. Replacing Q by amn and c_2Q by kmn in (2), we obtain

$$(3) \quad \pi = mn(ap - ac_1 - k)$$

The reason for writing profit in this form will become clear in subsequent sections when specific conditions for cooperation are introduced.

4. M - W₁ - N Structure: No Advertising

Suppose now that an intermediary level, W , exists between M and N . The middlemen are in contact with the producers and with the consumer. The middlemen now pick up the communications cost, but for each unit of product sold they receive a markup from the producers. If there are w middlemen in W , total communication costs incurred by level W are $w(k_1m + k_2n)$. Excess profit is defined as revenue minus cost minus a normal profit (that is, a normal return on investment). If commodity flow remains Q , revenue for intermediaries is pQ ; given assumption (x) they have to pay $(pQ - kmn)$ to the producers; their own costs are $w(k_1m + k_2n)$; and a normal return on investment is rF , for each intermediary. Therefore, excess profit π_E can be written as

$$(4) \quad \pi_E = kmn - w(k_1m + k_2n + rF)$$

Assuming that competition among intermediaries is such as to drive π_E to zero, the equilibrium number of middlemen in the VMS, w^* will be

$$(5) \quad w^* = kmn / (k_1m + k_2n + rF)$$

Let b = the number of units sold per communication link $W - N$.³

Q_1 = commodity flow through the $M-W-N$ structure

The number of $W-N$ links is wn . Therefore, we have

$$Q_1 = b w n$$

Expression (5) was arrived at assuming $Q_1 = Q$. Suppose now that

$Q_1 \neq Q$. Profit for producers is then π_1 ,

$$(6) \quad \pi_1 = b w n (p - c_1 - c'_2)$$

where

c'_2 = markup paid by a producer to a middleman for each unit sold by the latter to a consumer

Given the condition for cooperation stated in assumption (x), π_1 has to be equal to π . From the equality of π and π_1 we can derive c'_2 or simply $b w n c'_2$, the total amount that level M is willing to pay level W for performing the communication function. Equating (6) and (3) and solving for $b w n c'_2$, we obtain

$$(7) \quad b w n c'_2 = kmn - n(am - bw)(p - c_1)$$

Given that $p - c_1 > 0$, the total amount that producers are willing to pay to intermediaries will be less than, equal to or greater than kmn depending on whether $am - bw$ is positive, zero or negative. More will be said about that below. Using (7) excess profit can now be written as

$$(8) \quad \pi_E = kmn - n(am - bw)(p - c_1) - w(k_1 m + k_2 n + r F)$$

bw as a function of w

Let us now examine why in fact bw , sales per potential consumer, may be expected to vary with w . Suppose first that $w = 1$. Given that

consumers buy only from intermediaries, we can say that with $w = 1$, the product is not easily available, and consumers have to spend a considerable amount of effort in order to obtain the product. As more middlemen enter the VMS, the product becomes available at more sales outlets and it becomes easier for the consumer to shop for the product. Making the product more easily available creates demand. Thus, we may assume that bw is an increasing function of w . However, no matter how many middlemen there are, commodity flow will not become infinite, but will approach some limit. So, bw increases with w but at a decreasing rate.⁴ Let $(1 + d) am$ be the value which bw approaches when w becomes large.

Suppose that when there are no intermediaries the product is not easily available. Then, having a large number of intermediaries will increase demand, so that d will be positive. On the other hand, suppose that even without intermediaries the product is easily available. Also suppose that direct contact between producers and consumers is important for selling the product. Then, even with w large, commodity flow through $M - W - N$ may be less than what it would be through an $M-N$ structure, and d would be negative. It remains to be examined at what rate bw will increase with w . This rate will depend on geographic dispersion.

Suppose that the members of the VMS under consideration are located in a very small geographical area (and thus there is little dispersion), and that $w = 1$. Adding new intermediary firms does not make the product much more easily available. Even with only one intermediary firm, commodity flow should be close to $(1 + d) am$. The converse is true when the VMS covers a large geographical area, i.e. members in the VMS are

very dispersed. If there is only one middleman to serve consumers in such a VMS, commodity flow will be far away from its asymptotic value, and it will take many intermediaries before commodity flow comes close to $(1 + d)am$. The factors influencing bw can be summarized in the following functional relationship

$$(9) \quad bw = (1 + d) am \left[ew / (1 + ew) \right]$$

where e is a measure of the degree of dispersion. For $w = 0$, the right hand side of (9) is zero. This is as it should be. Indeed, assuming a VMS of the M-W-N type, no commodity flow can exist if level W is empty. It is easily verified that bw in (9) is an increasing function of w but with decreasing returns and that bw approaches $(1 + d)am$ as w gets large. If there is little dispersion, e is large. If e is large, $ew/(1 + ew)$ changes very little with w , and bw is close to $(1 + d)am$ even for small values of w . For a very dispersed market, e is very small, and w has to be much larger before bw comes close to $(1 + d)am$.

In summary then, expression (9) possesses all characteristics which we wanted bw to have.

Excess profit when bw is given by (9)

Incorporating w in the expression for excess profit (8), gives

$$\begin{aligned} \pi_E = & kmn - n \left[am - (1 + d) amew/(1 + ew) \right] (p - c_1) \\ & - w(k_1m + k_2n + rF) \end{aligned}$$

which reduces to

$$(10) \quad \pi_E = kmn - amn(1 - dew)(p - c_1) / (1 + ew) - w(k_1m + k_2n + rF)$$

Deriving the equilibrium value of w is straightforward, but the expression is complex. However, we do not need that result here. The main reason for developing this section was to set the stage for the next section where (9) will be used and where we will want to make comparisons between the $M - W - N$ structure with advertising and the $M - W - N$ structure without advertising.

5. $M - W_1 - N$ Structure: with advertising

In the previous section, we assumed that there was no advertising activity going on in the VMS. By advertising (through mass media or other means of communication), producers restore the direct link with ultimate consumers, which existed in the VMS without middlemen. Communication between levels M and N will affect demand, and thus commodity flow, in two ways. One, when a consumer is more familiar with a product it is easier for a middleman to transform a communication with a potential consumer into a sale to that consumer. Also, by persuading the consumer that a particular product is worth acquiring, producers in fact force middlemen to carry and promote the product.⁵ Indeed, not carrying the product could then result in losing a consumer. This loss is not only the loss of sales of the particular product but also the loss of sales of other products which the consumer usually buys from the middleman. In addition to this indirect effect of $M - N$ advertising on W , producers can also communicate persuasive information to middlemen, and thus put direct pressure on them to promote the product.⁶ Note that k_1 and k_2 are costs incurred by middlemen; k_3 and k_4 are paid for by the producers.

We expect that increasing advertising expenditures results in increased commodity flow. Many authors have assumed a logistic curve or a Gompertz curve to represent sales as a function of advertising. See for example (8). These curves show increasing returns to scale, for low values of advertising and decreasing returns to scale for high values of advertising. Let $b_o w$ represent sales per potential consumer without advertising.⁷ $(b_{\max} - b_o)w$ is the maximum possible increase in commodity flow per link that can be obtained by advertising for a given value of w . In other words $b_{\max} w$ corresponds to market potential for a given w . We cannot say a priori whether k_3 is more effective than k_4 . This will depend very much on the product. Detergents and even cars can be easily promoted by mass media advertising. For this type of product k_3 is quite effective. Other products, such as computers for example, are subjected to much more specific technical questioning before a sale can take place. Television would be an inefficient method of advertising computers. So for products like computers k_3 would be much less effective than k_4 . We should observe here that tastes may change. For example, electronic office equipment requires k_4 type promotion, but style of office equipment has become an important factor, so that using television to promote it is becoming increasingly popular.

We will consider two cases:

- 1) The condition for cooperation is as stated in assumption (x), that is, profit for producers in the M - W - N with advertising VMS must be equal to the profit that producers would make in the simple M - N

structure. The objective of the producers is then to maximize sales, given that particular profit level and also subject to having an equilibrium VMS, that is, no intermediaries will want to enter or to leave W.⁸

2) Producers want to maximize profit rather than being satisfied with their current profit. In this case assumption (x) does no longer hold.

Case 1: Sales Maximization

Profit for the producers, π_2 say, now becomes

$$(11) \quad \pi_2 = bwn(p - c_1 - c'_2) - mk_3 - wmk_4$$

where, c'_2 is the markup offered by producers per unit sold by middlemen to consumers. The total amount that producers are willing to pay to middlemen for performing the communication function is therefore $bwnc'_2$. From assumption (x) it follows that π_2 will be equal to π , where π is given by equation (3). Equating (11) and (3) we have

$$bwn(p - c_1 - c'_2) - mk_3 - wmk_4 = mn(ap - ac_1 - k)$$

so that total markup is

$$(12) \quad bwnc'_2 = kmn - n(am - bw)(p - c_1) - mk_3 - wmk_4$$

If $bwnc'_2 = kmn$, w is given by (5). If $bwnc'_2$ exceeds kmn , w will be greater than the value given by (5). Indeed, $bw_1nc'_2 > kmn$ means that excess profit for a given value of w is greater than that given by (4), so that equilibrium will be reached with more middlemen being in the

VMS. The condition for $bwnc_2'$ to be larger than k_{mn} is

$$(13) \quad -n(am - bw)(p - c_1) - mk_3 - wmk_4 > 0$$

To examine condition (13), we need to know how bw varies as a function of w , k_3 and k_4 . As was mentioned above, bw consists of two parts: $b_0 w$ and $w(b - b_0)$. $b_0 w$ is sales per consumer without advertising, and $w(b - b_0)$ is the increase in sales per consumer as a result of advertising. b_0 itself is also function of w as was demonstrated in section (4). Furthermore the effect of k_3 also depends on w , since mass media advertising which reaches the consumer is not very effective if the product is not easily available.

Therefore, instead of using advertising expenditures k_3 , we use "effective advertising expenditures", which, through reasoning analogous to that leading to the definition of bw in (9), can be expressed as follows

$$(14) \quad (\text{effective } k_3) = k_3 e_1 w / (1 + e_1 w)$$

where e_1 is analogous to e in expression (9). However, e_1 does not only incorporate geographic dispersion, but also quality of the advertising effort. Let us first assume that $k_4 = 0$. The effect of advertising will be represented here, not by a logistic, but by a logarithmic expression. The reason for doing this is twofold: (I) A logistic is not a very tractable curve to manipulate, and (II) empirical research supports that in the relevant range of values for advertising expenditures a logistic can be well approximated by a logarithmic function. See for example (9) (12).

Sales per consumer due to M - N advertising can thus be written as

$$(15) \quad t_1 \ln \left[k_3 e_1^w / (1 + e_1^w) \right]$$

where

$$t_1 = \frac{\ln \left[k_3 e_1^w / (1 + e_1^w) \right]}{\ln \left[k_3 e_1^w / (1 + e_1^w) \right] + 1}$$

increases by one unit.

Suppose now that the k_3 link does not exist, and that producers exert promotional pressure on the intermediary level only. The cost of this can be associated with each individual middleman, that is k_4 is the promotional expenditure per middleman. However, doubling the number of middlemen and spending the same amount on promotion for each of these, will not necessarily double the increase in sales due to promotional expenditures. Again this is related to having decreasing returns to availability.⁹ Increased sales due to M - W_1 advertising can then be written as

$$(16) \quad t_2 w^g \ln k_4$$

where

$$t_2 = \frac{\ln k_4}{\ln k_4 + 1}$$

increases by one unit.

$$g = \text{a positive constant smaller than one}$$

(15) and (16) represent the effect on sales of k_3 and k_4 when expenditures on k_4 and k_3 respectively are zero. However, k_3 and k_4 are not independent, but interact. For example, if k_3 is very high, increasing

k_3 by one dollar will not increase sales very much, but neither will increasing k_4 . Let us represent the total advertising effect per consumer by the following expression which takes the dependency into account. Assuming $t_1 = t_2 = t$ we have

$$(17) \text{ Total advertising effect} = t \ln \left[k_3 e_1 w / (1 + e_1 w) + k_4 w^g \right]$$

where

t = the increase in sales per link when

$$\ln \left[k_3 e_1 w / (1 + e_1 w) + k_4 w^g \right]$$

increases by one unit.

If $k_3 = 0$, (17) reduces to (16). If $k_4 = 0$, (17) reduces to (15).

Combining (9) and (17), sales per consumer as a function of w becomes

$$(18) \quad bw = (1 + d) am ew / (1 + ew) + t \ln \left[k_3 e_1 w / (1 + e_1 w) + k_4 w^g \right]$$

Recall that the total markup paid by producers to intermediaries, namely, $bwnc'_2$, was obtained in (12). Using (18) for bw in (12), $bwnc'_2$ can be rewritten as

$$\begin{aligned} bwnc'_2 = & kmn + n \left\{ am(ewd - 1) / (1 + ew) \right. \\ & + t \ln \left[k_3 e_1 w / (1 + e_1 w) + k_4 w^g \right] \left. \right\} (p - c_1) \\ & - mk_3 - wmk_4 \end{aligned}$$

Excess profit π_E is

$$\pi_E = bwnc'_2 - w(k_1 m + k_2 n + rF)$$

The objective is to maximize sales, that is, bwn , where bw is given by (18). At the same time we want to assure that the number of middlemen will not change. So we add excess profit equal to zero as a constraint.

This constraint also incorporates the minimum profit requirement.

The sales maximization problem can thus be written as follows

$$(19) \max_{k_3, k_4, w} n \left\{ (1 + d) \frac{amew}{(1 + ew)} + \ln \left[\frac{k_3 e_1 w}{(1 + e_1 w)} + k_4 \right] \right\}$$

subject to

$$k_{mn} + n \left\{ \frac{am(ewd - 1)}{(1 + ew)} + \ln \left[\frac{k_3 e_1 w}{(1 + e_1 w)} + k_4 \right] \right\} (p - c_1) - mk_3 - wmk_4 - w(k_1 m + k_2 n + rF) = 0$$

$$w \geq 1$$

$$k_3, k_4 \geq 0$$

The condition $w \geq 1$ is needed because for $w < 1$, the intermediary level W does not exist. The decision variables in the model are k_3 , k_4 and w , and by solving the problem above optimum values k_3^* , k_4^* and w^* for these variables are obtained. From a behavior point of view, producers decide only on values for k_3 and k_4 , and not on w_1 . However, if they set k_3 equal to k_3^* and k_4 equal to k_4^* , where k_3^* and k_4^* are optimal in the problem above, equilibrium in the VMS will be attained, that is, excess profit will be zero when the number of intermediaries is w^* , where w^* is the optimal value for w in the problem. Markup per unit c_2' does not enter the model explicitly. Yet, c_2' is also a decision variable, but one which is completely determined once the values for k_3 , k_4 , and w_1 are known. Indeed in the optimum solution, excess profit is zero and total markup bw^*nc_2' is equal to $w^*(k_1 m + k_2 n + rF)$. Markup per unit is then found by dividing $w^*(k_1 m + k_2 n + rF)$ by sales volume bw^*n , that is, the value of the objective function at optimality.

Note that the value of the objective function must be larger than Q , the amount of commodity flow in the $M - N$ type VMS. If not, producers would prefer to sell directly to consumers, rather than through intermediaries.

A discussion of the implications and usefulness of this model is postponed until section 7. Before that we will set up the profit maximization model, and then present a numerical example in section 6.

Case 2: Profit Maximization

Profit for producers is equal to the number of units sold times profit per unit, minus advertising expenditures. The number of units sold is bw . Profit per unit is $(p - c_1 - c'_2)$ where p is the sales price per unit, c_1 is average unit variable cost of production and c'_2 is markup per unit offered by producers to intermediaries. Total markup is then, $nbwc'_2$, and level W will be in equilibrium when excess profit is zero, that is, when $nbwc'_2 - w(k_1m + k_2n + rF) = 0$. The profit maximization problem is then

$$(20) \quad \max_{k_3, k_4, w, c'_2} \quad n \left\{ (1 + d) \frac{amew}{(1 + ew)} + t \ln \left[\frac{k_3 e_1 w}{(1 + e_1 w) + k_4 w^g} \right] \right\} (p - c_1 - c'_2) - mk_3 - wmk_4$$

subject to

$$\begin{aligned} & n \left\{ (1 + d) \frac{amew}{(1 + ew)} + t \ln \left[\frac{k_3 e_1 w}{(1 + e_1 w) + k_4 w^g} \right] \right\} c'_2 \\ & - w(k_1m + k_2n + rF) = 0 \end{aligned}$$

$$w \geq 1$$

$$c'_2, k_3, k_4 \geq 0$$

In the profit maximization case, markup c'_2 is an active decision variable, in contrast to the sales maximization case, where c'_2 was determined by the other decision variables. If there are no intermediaries, producers make a profit of $Q(p - c_1 - c_2)$ as we found in (2). In order for the M - W - N with advertising to be preferable for the producers, the value of the objective function (20) at optimality must exceed $Q(p - c_1 - c_2)$. Again producers determine only the value for k_3 , k_4 , and c'_2 and not for w . Solving the model determines optimal values k_3^* , k_4^* , $c_2'^*$, and w_1^* . Then when producers set k_3 , k_4 , and c'_2 equal to k_3^* , k_4^* , and $c_2'^*$ respectively, the number of intermediaries when excess profit is zero will equal w_1^* .

6. Numerical Example

In this section an example illustrating the sales maximization and profit maximization models developed in section (5) is studied. Fiacco and McCormick's SUMT program was used to solve the problems [10].

Let	$m = 10$	$n = 10$	$p = 10$	$c_1 = 5$
	$t = 10$	$k = 2$	$k_1 = 1$	$k_2 = 1$
	$r = .10$	$F_{w_1} = 200$	$d = 2$	$e = .5$
	$e_1 = .5$	$Q = 100$	$g = .5$	

Sales per link in the M - N type VMS is then $a = Q/mn = 1$

Cost of communication is $k_{mn} = 2 \times 10 \times 10 = 200$

Cost of communication is also $c_2 Q$, so that cost of communication per unit is $c_2 = k_{mn}/Q = 200/100 = 2$

And profit for producers, π , if they sell directly to consumers is

$$\pi = Q(p - c_1 - c_2) = 100(10 - 5 - 2) = 300$$

Now let us examine what would happen if an intermediary level, W , exists. The sales maximization problem as stated in (19) reduces to

$$\begin{aligned} \max \quad & 150W / (1 + .5w) + 10 \ln \left[.5k_3w / (1 + .5w) + k_4^{w \cdot 5} \right] \\ \text{s.t.} \quad & 200 + (500w - 500) / (1 + .5w) + 500 \ln \left[.5k_3w / (1 + .5w) \right. \\ & \left. + k_4^{w \cdot 5} \right] - 10k_3 - 10k_4w - 40w = 0 \\ & k_3, k_4 \geq 0 \quad \quad \quad w \geq 1 \end{aligned}$$

The following optimum values are obtained:¹⁰

$$k_3^* = 8.559; \quad k_4^* = 29.593; \quad w^* = 31.226$$

At optimality, the value of the objective function, that is, the level of sales is 2,174.9 compared to $Q = 100$, in the $M - N$ type VMS.

With excess profit equal to zero total markup, $bwnc_2'$, equals $w(k_1m + k_2n + rF)$. So that we can write $bw^*nc_2'^* = 40w^* = 40(31.226) = 1,249.04$. Maximum sales is bw^*n , the objective function value. Therefore, optimal markup is $c_2'^* = 1,249.04 / 2,174.9 = .574$

Profit earned by producers is now

$$bw^*n(p - c_1 - c_2'^*) - 10k_3^* - 10k_4^{**}w^*$$

$$\begin{aligned}
 &= 2,174.9(10 - 5 - .574) - 10(8.559) - 10(29.593)(31.226) \\
 &= 9,626.1 - 85.59 - 9240.51 \\
 &= 300
 \end{aligned}$$

Profit is again 300, as it was in the VMS without intermediaries. That is, of course, as it should be, since in setting up the sales maximization model we assumed that assumption (X) holds.

Suppose now that producers want to maximize profit. Using the profit maximization model developed in (20) we obtain

$$\begin{aligned}
 &\max 10 \left\{ 15w / (1 + .5w) + 10 \ln \left[.5k_3^w / (1 + .5w) + k_4^w \right]^{.5} \right\} \\
 &\quad (5 - c_2') - 10 k_3 - 10k_4w \\
 \text{s.b.} \quad &10 \left\{ 15w / (1 + .5w) + 10 \ln \left[.5k_3^w / (1 + .5w) + k_4^w \right]^{.5} \right\} c_2' - 40w = 0 \\
 &w \geq 1 \\
 &c_2', k_3, k_4 \geq 0
 \end{aligned}$$

The following optimal values are obtained

$$k_3^* = 0; k_4^* = 9.067; w^* = 28.936; c_2'^* = .789$$

The value of the objective function, that is, maximum profit, is then 3,551.5,

compared to 300 as was obtained in the M-N structure or in the sales maximization case.

Excess profit at optimality is zero, so that $bnw^* c_2'^* = 40w^*$
 Sales volume is then $bnw^* = 40w^* / c_2'^* = 40 \times 28.936 / .789 = 1,466.9$,

compared to 100 for the direct distribution case and 2,174.9 in the sales maximization case.

Both models were also run for $g = .2$ instead of $.5$. The results for both cases are summarized in table 1.

TABLE 1
COMPARISON OF SALES MAXIMIZATION VERSUS
PROFIT MAXIMIZATION

	g	Direct Distribution	Sales Maximization	Profit Maximization
Profit	.5	300	300	3,551.5
	.2	300	300	1,365.87
Sales	.5	100	2,174.9	1,466.9
	.2		786.93	779.0
k_3^*	.5		8.559	0
	.2		155.7	191.35
k_4^*	.5		29.593	9.067
	.2		6.628	0
w_1^*	.5		31.226	28.936
	.2		19.547	15.339
c_2^*	.5		.574	.789
	.2		.993	.7877

7. Summary and Conclusion

Vertical market structures with one intermediary level were examined where sales volume is not a given constant, but depends on the number of intermediaries in that level, on consumer advertising and on intermediary advertising. In trying to determine the level of advertising expenditures which will optimize their objective, little attention is

usually paid to the fact that there is also an optimum value for the markup which producers offer to intermediaries for each unit sold. Similarly, behavior of intermediaries is usually not taken into consideration. In this paper we presented two models where markup as a decision variable and behavior of intermediaries were explicitly taken into account. In the first model, the objective is to maximize sales, given a certain minimum profit, and subject to having equilibrium in the VMS. Optimal values for k_3 , k_4 , and w are then determined by the model. With these values, the optimal value for markup c_2' can be calculated. The number of intermediaries is integrated in the decision model without really being within the control of the producers. But if producers set k_3 , k_4 , and c_2' equal to the optimal values determined by the model, the number of intermediary firms w making excess profit equal to zero will be equal to the optimum value of w given by the model. In the second model, the objective is to maximize profit subject to having equilibrium in the VMS. The model determines optimal values for k_3 , k_4 , c_2' and w . Again producers set k_3 , k_4 , and c_2' equal to these optimal values and w then adjusts in such a way that it reaches its optimal value at equilibrium.

With regard to the practical use of these models, we should recall the simplicity of some of the underlying assumptions. Some of these assumptions could fairly easily be relaxed. For example, k_1 and k_2 as functions of w would add some complexity to the models but does not present a real problem. Relaxing some other assumptions would add considerably more difficulties. An example of this would be to try to include product differentiation and cross elasticities of advertising.

Another difficulty arises from the fact that middlemen in this paper are rather passive agents: they are satisfied with some minimum return on their investment and additional middlemen enter the market until excess profit is zero. More complicated intermediary behavior is of course possible.

In short then, this paper derives its primary importance from the following: In most of the literature on VMS either equilibrium behavior is studied without consideration of objectives or some objective function is being optimized without taking into account behavior of intermediaries. This paper is a modest attempt to bring the two approaches together. At the same time an often neglected factor namely, markup offered by producers to intermediaries, was included in the model as a decision variable.

Footnotes

- ¹For a detailed analysis of Baligh and Richartz' extended Balderston model see [11, Chapter 2].
- ²On rebates see [1, pp. 152-161] [4, pp. 31-33] and [11, pp. 37-38 and pp. 57-62].
- ³ b will be a function of w . The functional form will be discussed later in this section.
- ⁴Note that bw could be increasing for a while then decreasing.
- ⁵This is a commonly held belief. Haines and Silk have recently casted some doubts [7].
- ⁶Consumer advertising by intermediaries is not considered here.
- ⁷ b_o is the b used in section 4, that is $(1 + d) a_m e / (1 + e w)$.
- ⁸Baumol has suggested that managers may indeed seek to maximize sales and not profits, subject only to a minimum acceptable profit constraint [5] [6]. Of course this minimum acceptable profit could be different from that corresponding to assumption (X).
- ⁹Returns could also be increasing first, then decreasing.
- ¹⁰The program was run several times using different sets of input data, in order to decrease the possibility of ending up with a local optimum.

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